How Mesoscopic Staircases Condense to Macroscopic Barriers

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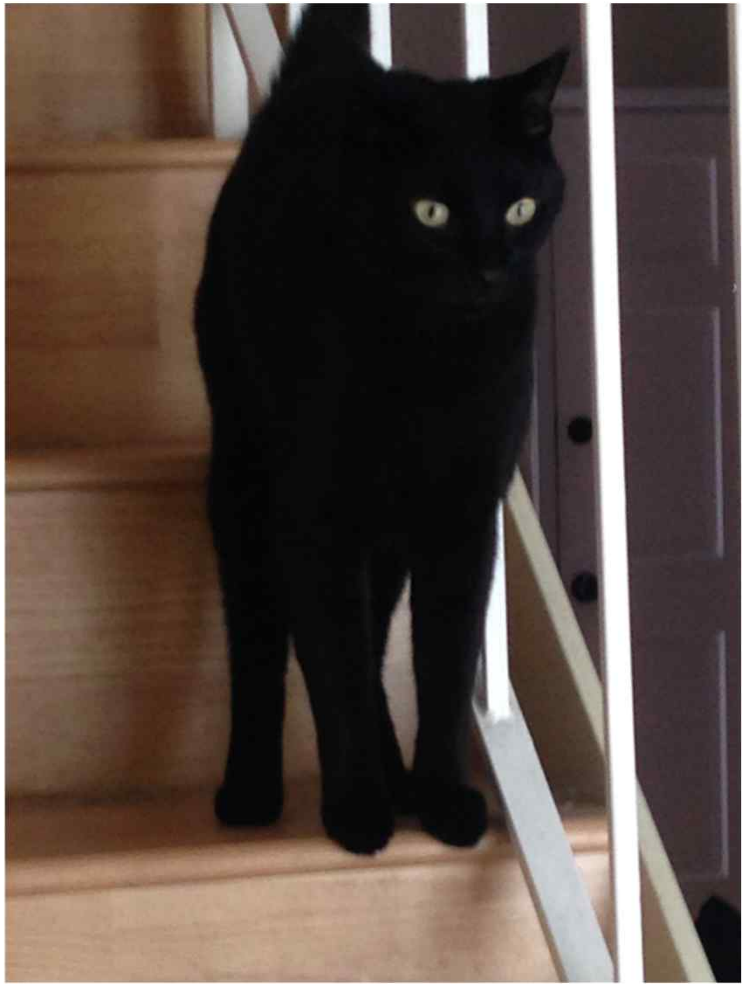


Some Questions

I)

Re: Drift-ZF Turbulence

- Impact of ZF well established
- Effectively linear modulation theory developed

But:

- What sets scale of ZF field? → V_E'
- How does modulational instability evolve nonlinearly, saturate
- N.B.: Predator-Prey feedback channel
- Saturation ←→ scale connection?

Re: Barrier

– ZF/Flow shear → barrier connection?

– I-phase → LCO → transport bifurcation study is ~ 0D

– Mesoscale → Macroscale coupling in barrier transitions?

– Mechanisms of ‘non-locality’?

Outline

- The Questions
- The ExB staircase
- Beyond the color VG – a model:

  Hasegawa – Wakatani staircase

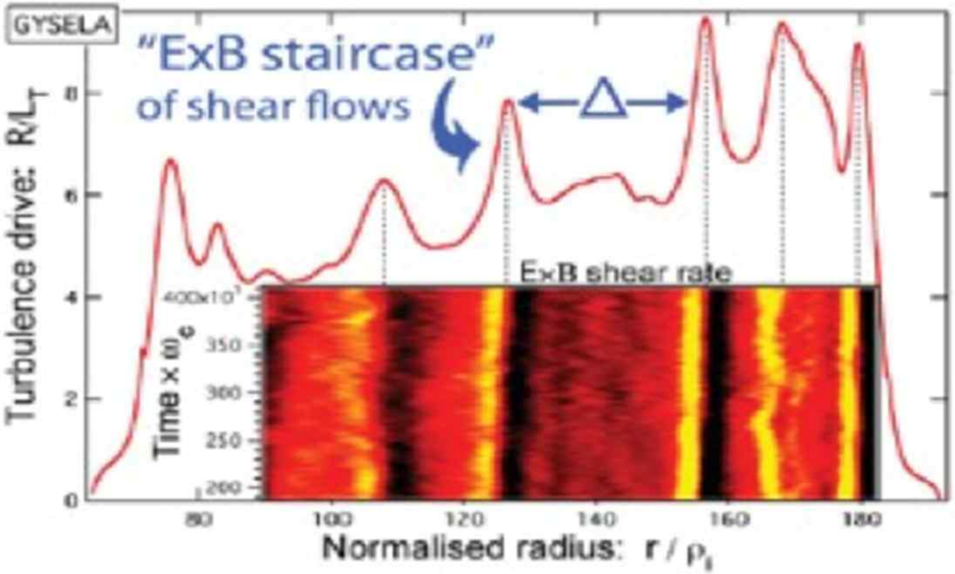
- Global transport bifurcations via condensation
- Some ideas for future study

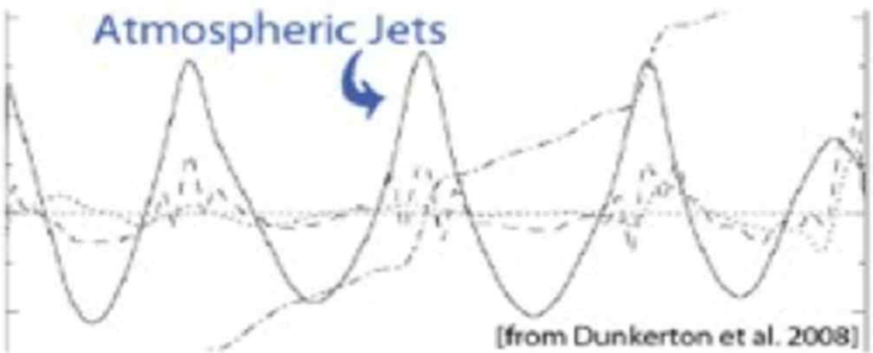
Motivation: ExB staircase formation

- ExB flows often observed to self-organize in magnetized plasmas

  eg. mean sheared flows, zonal flows, ...

- \`ExB staircase' is observed to form (G. Dif-Pradalier, P.D. et al. Phys. Rev. E. '10)





- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases
- so-named after the analogy to PV staircases and atmospheric jets
- Step spacing → avalanche outer-scale

- Interesting as:
  - Clear scale selection
  - Clear link of:
  - ZF scale ←→ avalanche scale → corrugation

But:

  - Systematic scans lacking
  - Somewhat difficult to capture

- Need a <u>MODEL</u>

The Hasegawa-Wakatani Staircase:

Profile Structure:

From Mesoscopics → Macroscopics

H-W Drift wave model – Fundamental prototype

- Hasegawa-Wakatani : simplest model incorporating instability

$$V = \frac{c}{B}\hat{z} \times \nabla \phi + V_{pol}$$

$$J_\perp = n|e|V^i_{pol} \qquad \eta J_\parallel = -\nabla_\parallel \phi + \nabla_\parallel p_e$$

$$\nabla_\perp \cdot J_\perp + \nabla_\parallel J_\parallel = 0$$ → vorticity: $\rho_s^2 \frac{d}{dt}\nabla^2 \phi = -D_\parallel \nabla_\parallel^2 (\phi - n) + \nu \nabla^2 \nabla^2 \phi$

$$\frac{dn_e}{dt} + \frac{\nabla_\parallel J_\parallel}{-n_0|e|} = 0$$ → density: $\frac{d}{dt} n = -D_\parallel \nabla_\parallel^2 (\phi - n) + D_0 \nabla^2 n$

→ PV conservation in inviscid theory $\frac{d}{dt}\left(n - \nabla^2 \phi\right) = 0$

→ PV flux = particle flux + vorticity flux

→ zonal flow being a counterpart of particle flux

QL: $\frac{\partial}{\partial t}\langle n \rangle = -\frac{\partial}{\partial r}\langle \tilde{\upsilon}_r \tilde{n} \rangle$

→? $\frac{\partial}{\partial t}\langle \nabla^2 \phi \rangle = -\frac{\partial}{\partial r}\langle \tilde{\upsilon}_r \nabla^2 \tilde{\phi} \rangle = -\frac{\partial^2}{\partial r^2}\langle \tilde{\upsilon}_r \tilde{\upsilon}_\theta \rangle$

- Hasegawa-Mima ($D_\parallel k_\parallel^2 / \omega >> 1 \;\rightarrow\; n \sim \phi$)

$$\frac{d}{dt}\left(\phi - \rho_s^2 \nabla^2 \phi\right) + \upsilon_* \partial_y \phi = 0$$

The Reduced 1D Model

Reduced system of evolution Eqs. is obtained from HW system for DW turbulence.

Variables:

$u = \partial_x V_y$ Zonal shearing field

Reduced density: $\log(N/N_0) = n(x,t) + \tilde{n}(x,y,t)$, **Vorticity:** $\rho_s^2 \nabla_\perp^2 (e\varphi/T_e) = u(x,t) + \tilde{u}(x,y,t)$

Potential Vorticity (PV): $q = n - u$, **Turbulent Potential Enstrophy (PE):** $\varepsilon = \frac{1}{2}\left\langle (\tilde{n} - \tilde{u})^2 \right\rangle$

Two components

Mean field equations:

density $\partial_t n = -\partial_x \Gamma_n + \partial_x [D_c \partial_x n]$, $\Gamma_n = \left\langle \tilde{v}_x \tilde{n} \right\rangle = -D_n \partial_x n$ → **Reflect instability**

$\text{Taylor ID}: \ \Pi_u = \left\langle \tilde{v}_x \tilde{u} \right\rangle = \partial_x \left\langle \tilde{v}_x \tilde{v}_y \right\rangle$

vorticity $\partial_t u = -\partial_x \Pi_u + \partial_x [\mu_c \partial_x u]$, $\Pi_u = \left\langle \tilde{v}_x \tilde{u} \right\rangle = (\chi - D_n)\partial_x n - \chi \partial_x u$

Residual vort. flux **Turb. viscosity**

Turbulence evolution: (Potential Enstrophy) **From closure**

$$\partial_t \varepsilon = \partial_x [D_\varepsilon \partial_x \varepsilon] - (\Gamma_n - \Gamma_u)[\partial_x (n - u)] - \varepsilon_c^{-1} \varepsilon^{3/2} + P$$

Turbulence spreading, Internal production, dissipation, External production $\sim \gamma\varepsilon$

Two fluxes Γ_n, Γ_u set model !

What is new in this model?

○ **In this model PE conservation is a central feature.**

○**Mixing of Potential Vorticity (PV) is the fundamental effect regulating the interaction between turbulence and mean fields. Mixing <u>inhomogeneous</u>**

○**Dimensional and physical arguments used to obtain functional forms for the turbulent diffusion coefficients. From the QL relation for HW system we obtain**

$$D_n \cong l^2 \frac{\varepsilon}{\alpha} \qquad \chi \cong c_\chi l^2 \frac{\varepsilon}{\sqrt{\alpha^2 + a_u u^2}}$$

* l **<u>Dynamic mixing length</u>** → **Rhines scale sets**

α **Parallel diffusion rate**

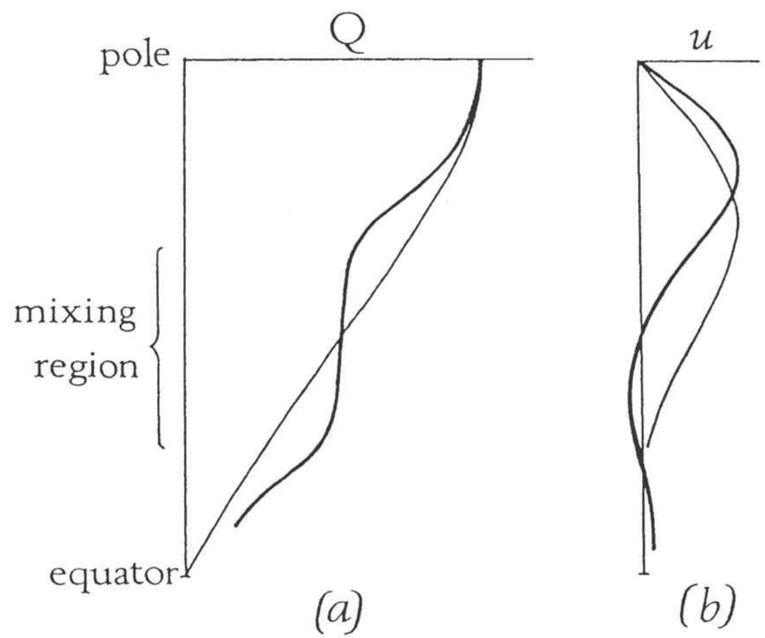
○***Inhomogeneous mixing of PV results in the sharpening of density and vorticity gradients in some regions and weakening them in other regions, leading to shear lattice and density staircase formation.***

Jet sharpening in stratosphere, resulting from inhomogeneous mixing of PV. (McIntyre 1986)

PV $Q = \nabla^2 \psi + \beta y$

$\nabla^2\psi$ → Relative vorticity

βy → Planetary vorticity



Perspective on (Rhines) Scale

- Note: $l^2 = \frac{1}{1+1/l_{Rh}^2} \rightarrow \frac{1}{1+\langle q \rangle'^2/\epsilon}$ $\quad (l_f \sim 1)$
- Reminiscent of weak turbulence perspective:

$$D = D_{pv} = \sum_{\vec{k}} \frac{\langle \tilde{V}^2 \rangle \Delta\omega_{\vec{k}}}{\omega_{\vec{k}}^2 + \Delta\omega_{\vec{k}}^2}$$

$$\omega_{\vec{k}} = -k_x \langle q \rangle' / k^2$$

$$\Delta\omega_{\vec{k}} \approx k\tilde{V}_{\vec{k}}$$

Ala' Dupree'67:

$$D_{pv} \approx \frac{1}{k^2}\left(\sum_{\vec{k}} k^2 \langle \tilde{V}^2 \rangle_{\vec{k}} - \frac{k_x^2 \left(\langle q \rangle'\right)^2}{(k^2)^2}\right)^{1/2}$$

Steeper $\langle q \rangle'$ quenches diffusion → mixing reduced via <u>PV gradient</u> feedback

$$D_{pv} \approx \frac{l_0^2 \, \epsilon^{\frac{1}{2}}}{1 + \frac{l_0^2}{\epsilon} (\langle q \rangle')^2} \quad \leftarrow$$

- ω vs $\Delta\omega$ dependence gives D_{pv} roll-over with steepening
- Rhines scale appears naturally, in feedback strength → roll over scale
- Recovers effectively same model

Physics:

① "Rossby wave elasticity' (MM) → steeper $\langle q \rangle'$ → stronger memory (i.e. more 'waves' vs turbulence)

→ ② Distinct from shear suppression → interesting to dis-entangle

Staircase structure

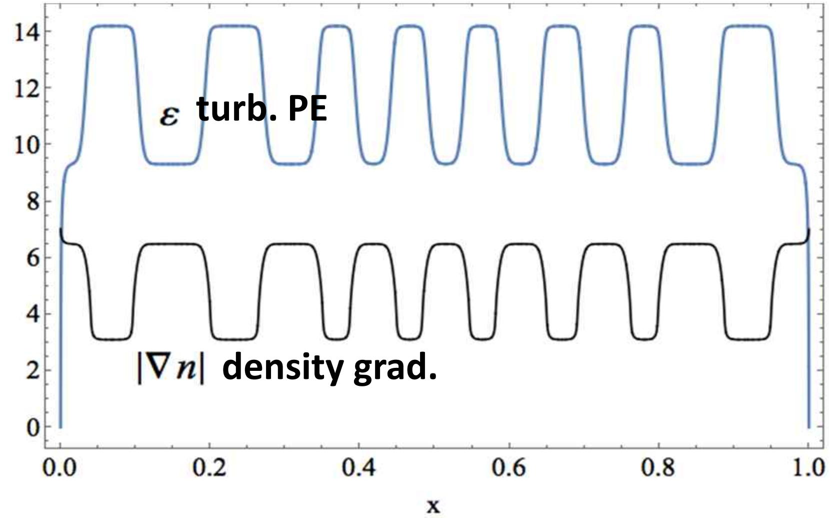
Snapshots of evolving profiles at t=1 (non-dimensional time)

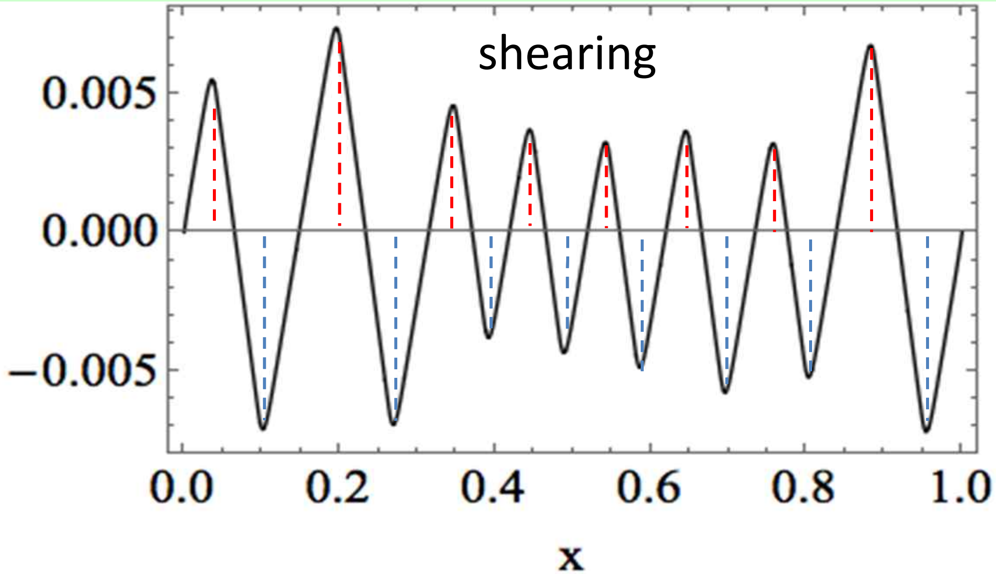
Initial conditions: $n = g_0(1-x), \quad u = 0, \quad \varepsilon = \varepsilon_0$

Boundary conditions: $n(0,t) = g_0,\ n(1,t) = 0; \quad u(0,1;t) = 0; \quad \partial_x \varepsilon(0,1;t) = 0$

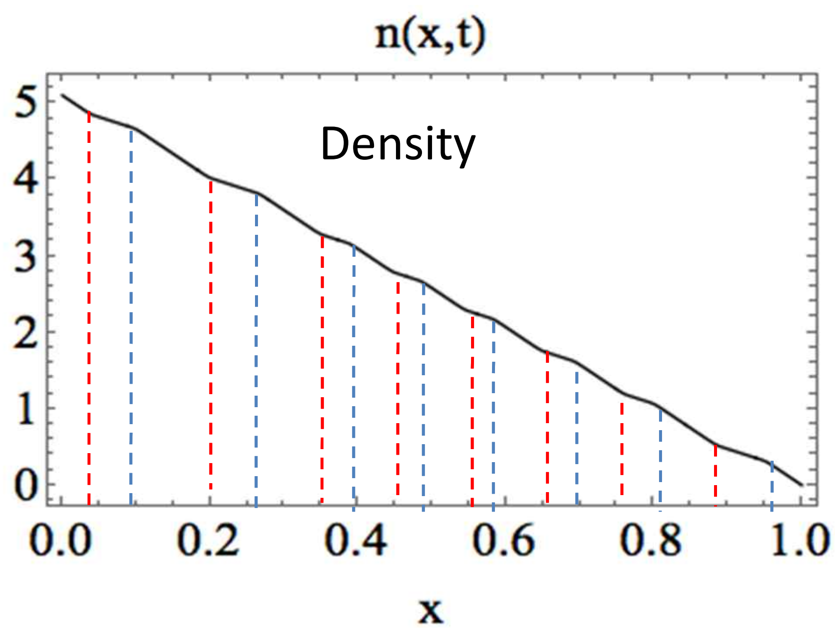
Structures:

- ○**Staircase in density profile:**
  - **jumps → regions of steepening**
  - **steps → regions of flattening**
- ○**At the jump locations, turbulent PE is suppressed.**
- ○**At the jump locations, vorticity gradient is positive**





Density + Vorticity lattices

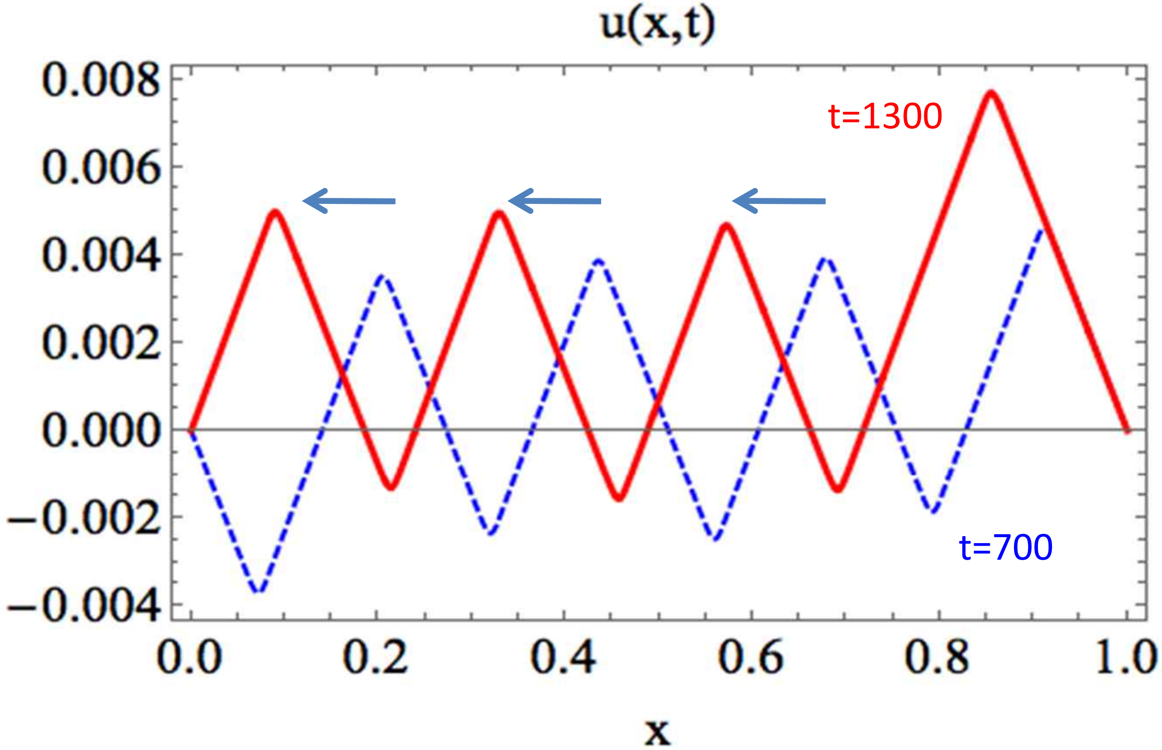


Dynamic Staircases

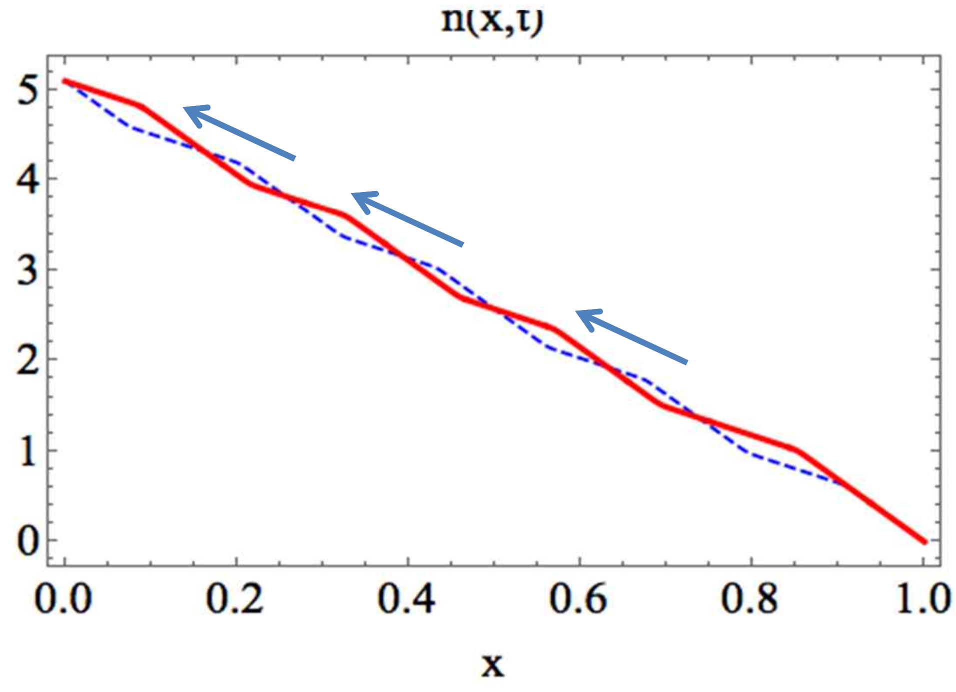
○Shear pattern detaches and delocalizes from its initial position of formation.

○Mesoscale shear lattice moves in the up-gradient direction. Shear layers condense and disappear at x=0.

○Shear lattice propagation takes place over much longer times. From t~O(10) to t~(10^4).



○Barriers in density profile move upward in an "<u>Escalator-like</u>" motion.



➔ **Macroscopic Profile Re-structuring**

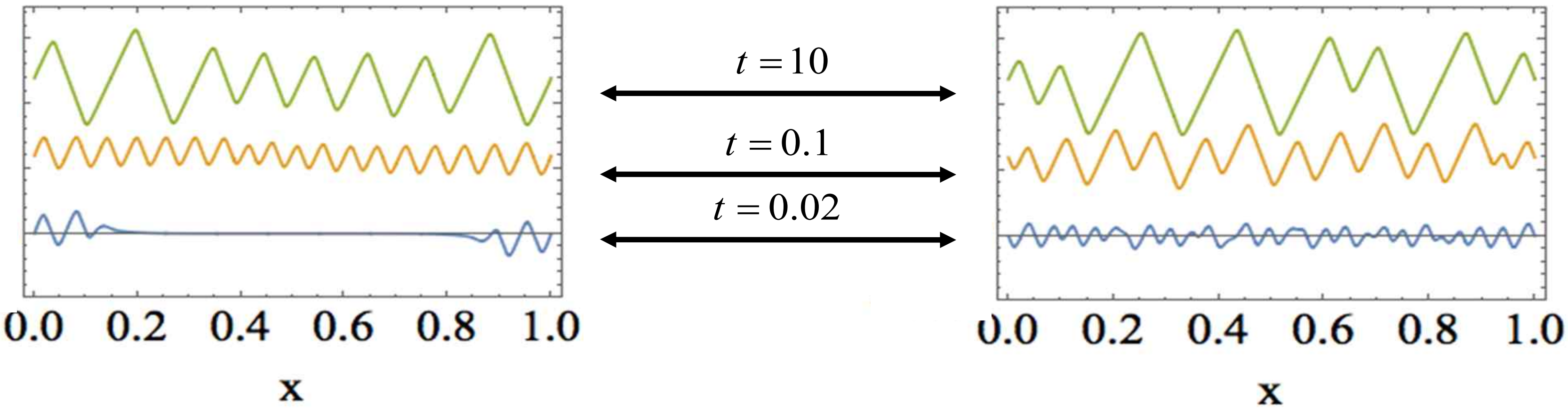
↕

'Non-locality'

Mergers Occur

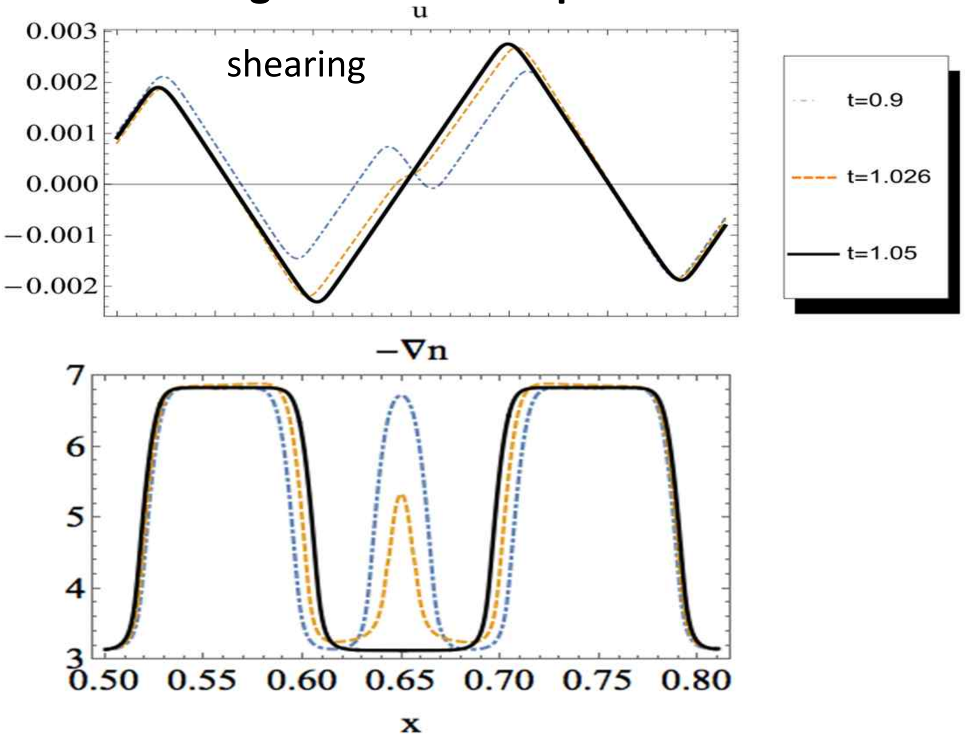
Nonlinear features develop from 'linear' instabilities

$\varepsilon(x=0,1)=0$ u(x,t) $\partial_x \varepsilon(x=0,1)=0$ u(x,t)

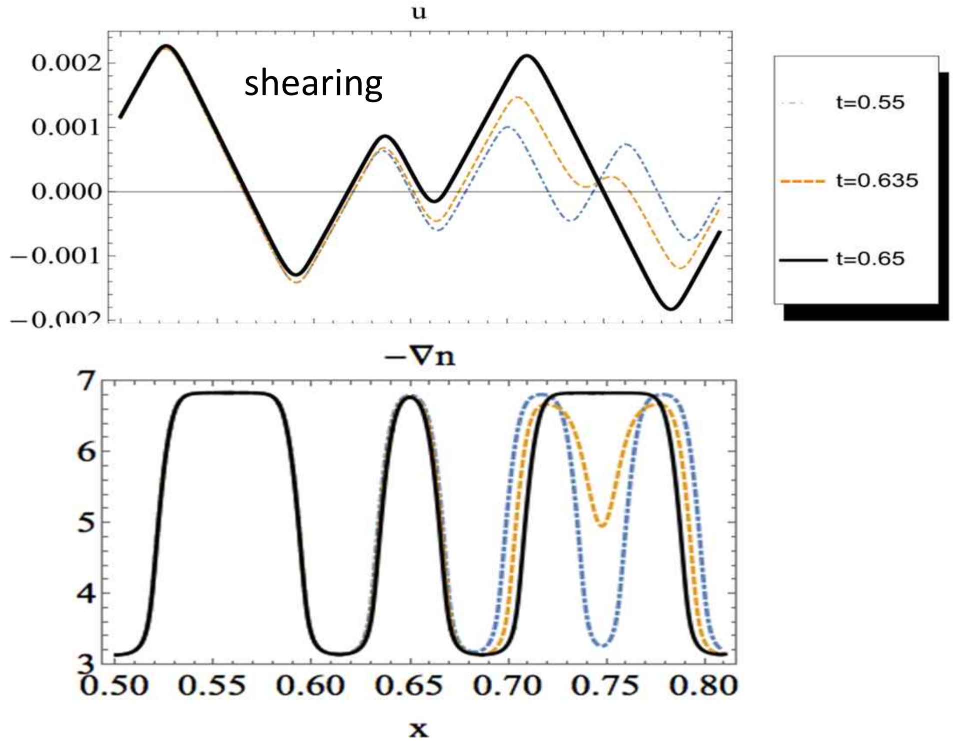


Local profile reorganization: Steps and jumps merge (continues up to times t~O(10))

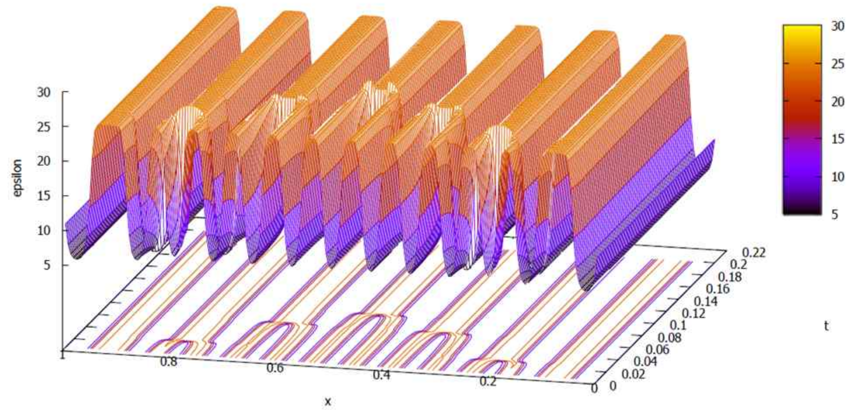
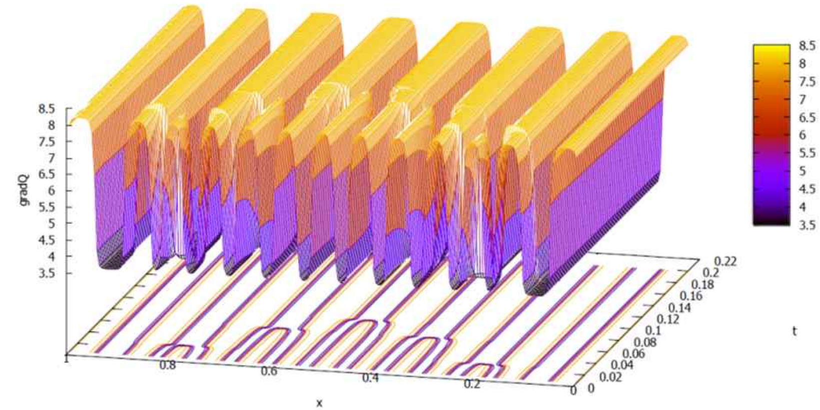
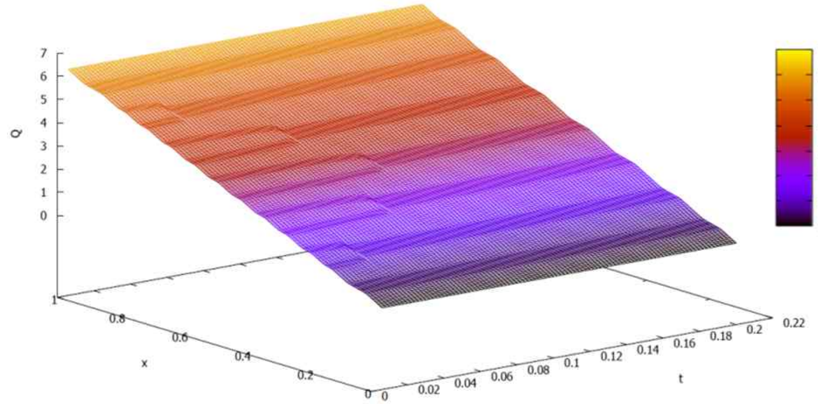
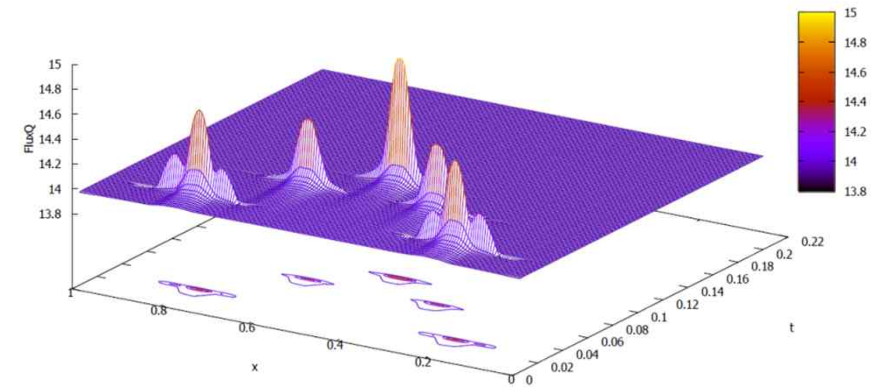
Merger between steps



Merger between jumps



Illustrating the merger sequence (QG-HM)

- ϵ
- Q_y

} top

- Q
- Γ_q

} bottom

Note later staircase mergers induce strong flux episodes!

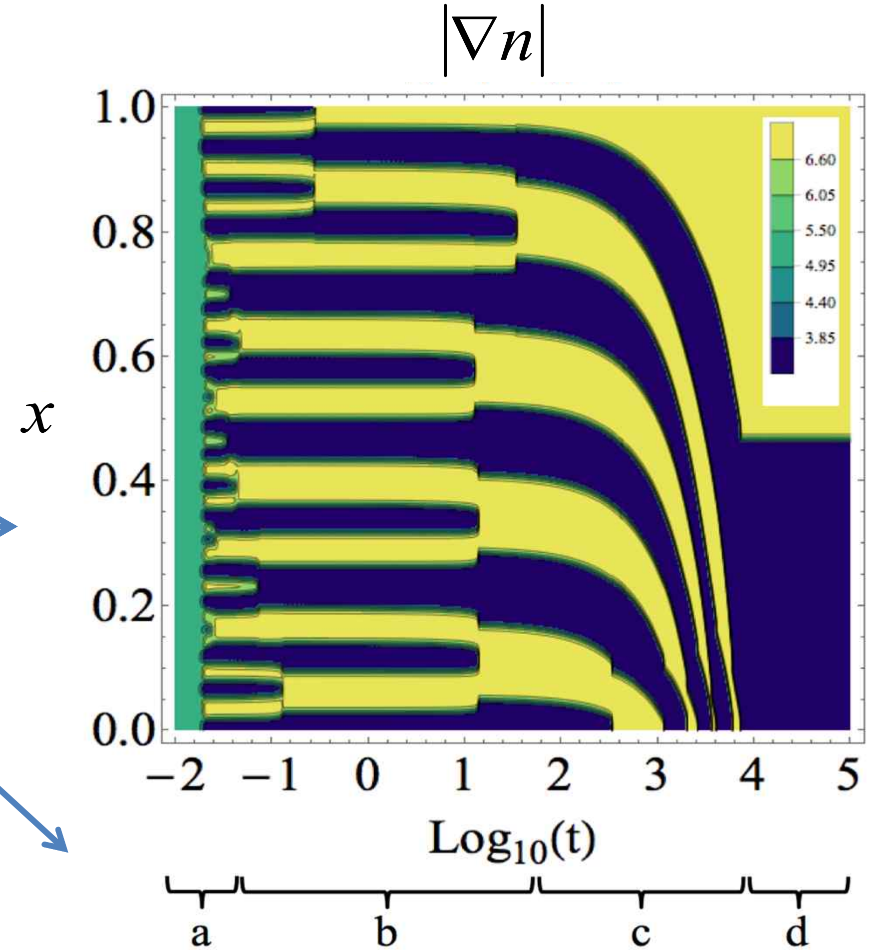
Time evolution of profiles

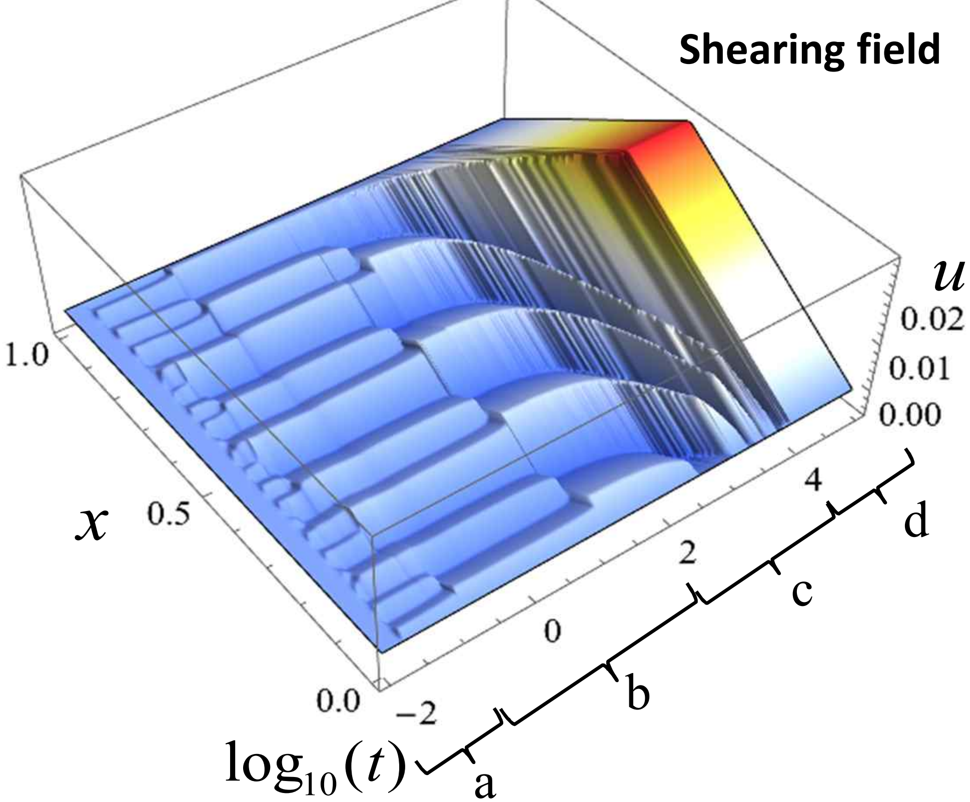
(a) Fast merger of micro-scale SC. Formation of meso-SC.

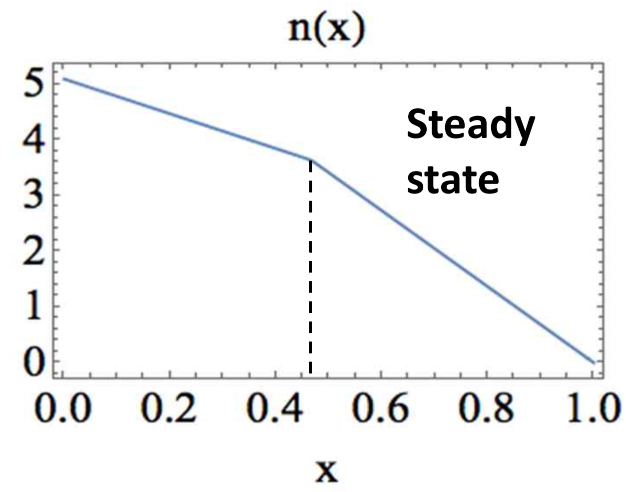
(b) Meso-SC coalesce to barriers

(c) Barriers propagate along gradient, condense at boundaries

(d) Macro-scale stationary profile







- The Point:
  - Macroscopic barrier emerges from hierarchical sequence of mergers and propagation, condensation
  - (Somewhat) familiar bi-stable transport model

  But

  - Barrier formation is NOT a local process!

  → Begs for flux driven, not BVP analysis!

Macroscopics: Flux driven evolution

We add an external particle flux drive to the density Eq., use its amplitude Γ_0 as a control parameter to study:

✓**What is the mean profile structure emerging from this dynamics?**

✓**Variation of the macroscopic steady state profiles with Γ_0 (shearing, density, turbulence, and flux).**

✓**Transport bifurcation of the steady state (macroscopic)**

✓**Particle flux-density gradient landscape.**

$$\partial_t n = -\partial_x \Gamma - \partial_x \Gamma_{dr}(x,t)$$

➔ **Write source as $\nabla \cdot \mathbf{\Pi_{ex}}$**

External particle flux (drive) $\quad \Gamma_{dr}(x,t) = \Gamma_0(t)\exp[-x/\Delta_{dr}]$

Internal particle flux (turb. + col.) $\quad \Gamma = -[D_n(\varepsilon,\partial_x q) + D_{col}]\partial_x n$

Transition to Enhanced Confinement can occur

Steady state solution for the system undergoes a transport bifurcation as the flux drive amplitude Γ_0 is raised above a threshold Γ_{th}.

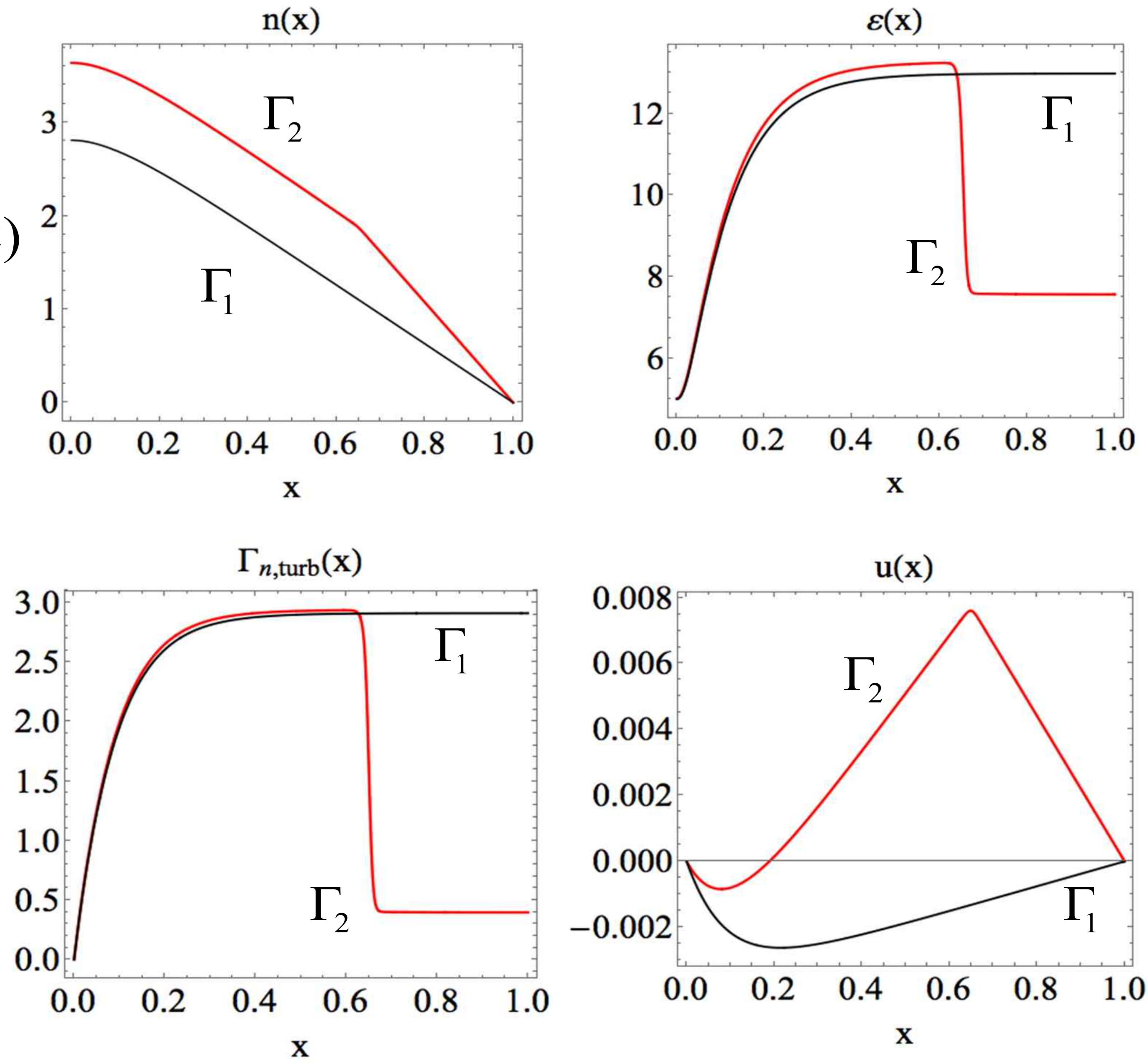
$$\Gamma_1 < \Gamma_{th} < \Gamma_2$$

$$\Gamma_0 = \Gamma_1 \rightarrow \text{Normal Conf. (NC)}$$

$$\Gamma_0 = \Gamma_2 \rightarrow \text{Enhanced Conf. (EC)}$$

With NC to EC transition we observe:

- Rise in density level
- Drop in turb. PE and turb. particle flux beyond the barrier position
- Enhancement and sign reversal of vorticity (shearing field)



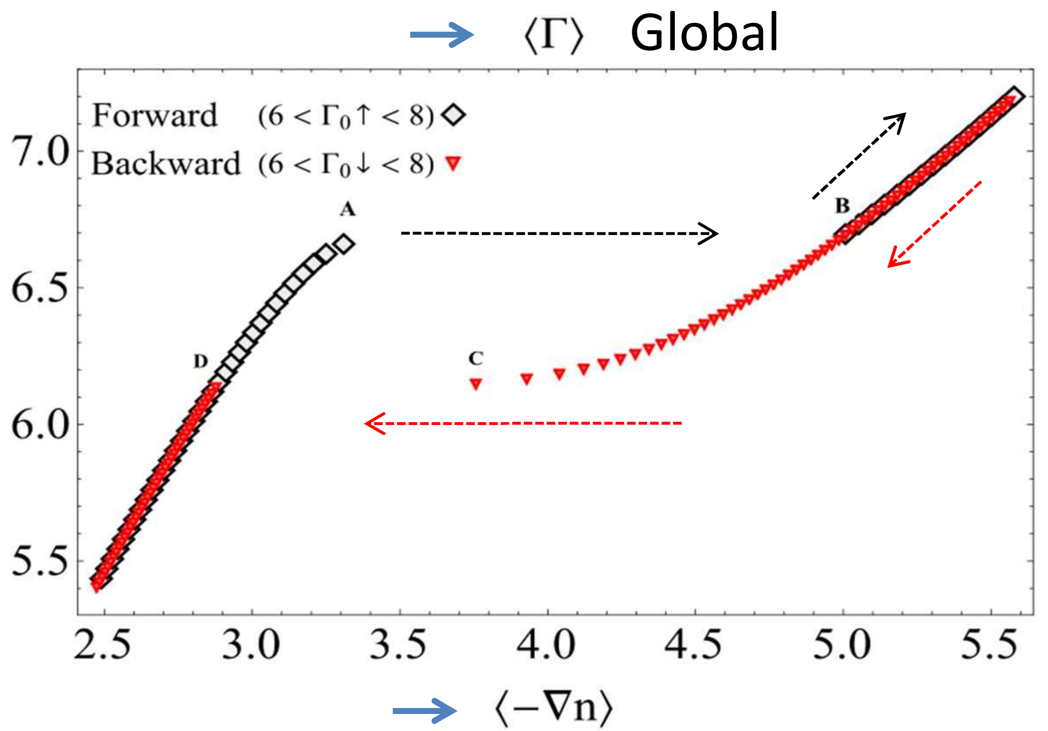
Hysteresis evident in the GLOBAL flux-gradient relation

In one sim. run, from initially flat density profile, Γ_0 is adiabatically raised and lowered back down again.

Forward Transition:
Abrupt transition from NC to EC (from A to B). During the transition the system is not in quasi-steady state.

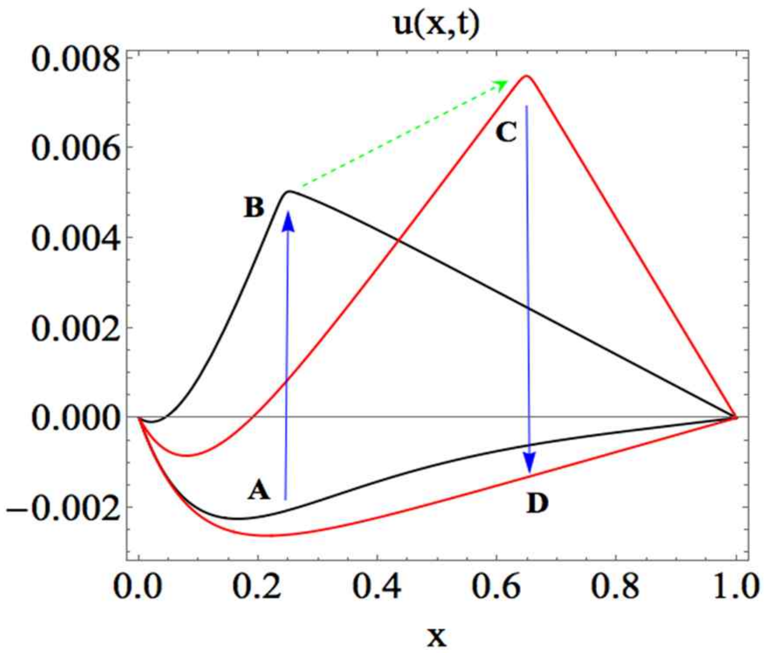
From B to C:
We have continuous control of the barrier position Barrier moves to the right with lowering the density gradient.

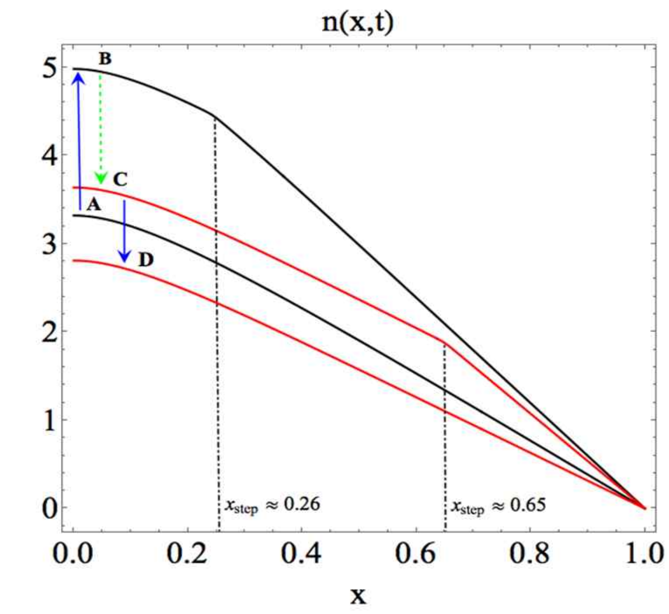
Backward Transition:
Abrupt transition from EC to NC (from C to D). Barrier moves rapidly to the right boundary and disappears. system is not in quasi-steady.



$$\langle\Gamma\rangle = \int_0^1 \Gamma(x)dx$$

$$\langle-\partial_x n\rangle = \int_0^1 [-\partial_x n(x,t)]dx$$



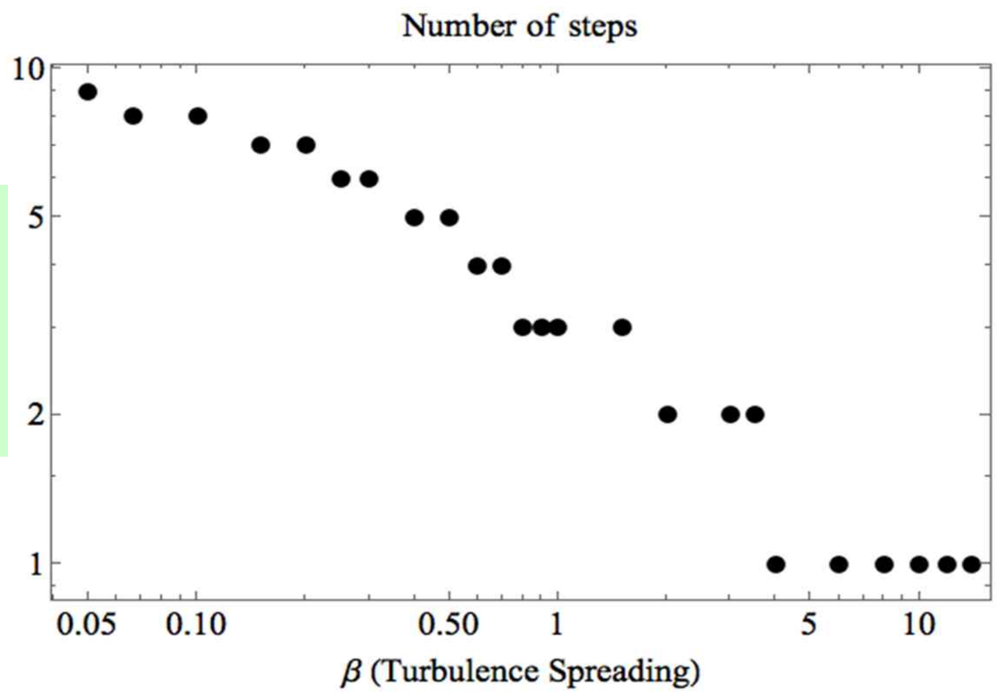


Role of Turbulence Spreading

○ **Large turbulence spreading wipes out features on smaller spatial scales in the mean field profiles, resulting in the formation of fewer density and vorticity jumps.**

$$\partial_t \varepsilon = \beta \partial_x [(l^2 \varepsilon^{1/2}) \partial_x \varepsilon] + \ldots$$

- $\beta \to 0$ excessive profile roughness



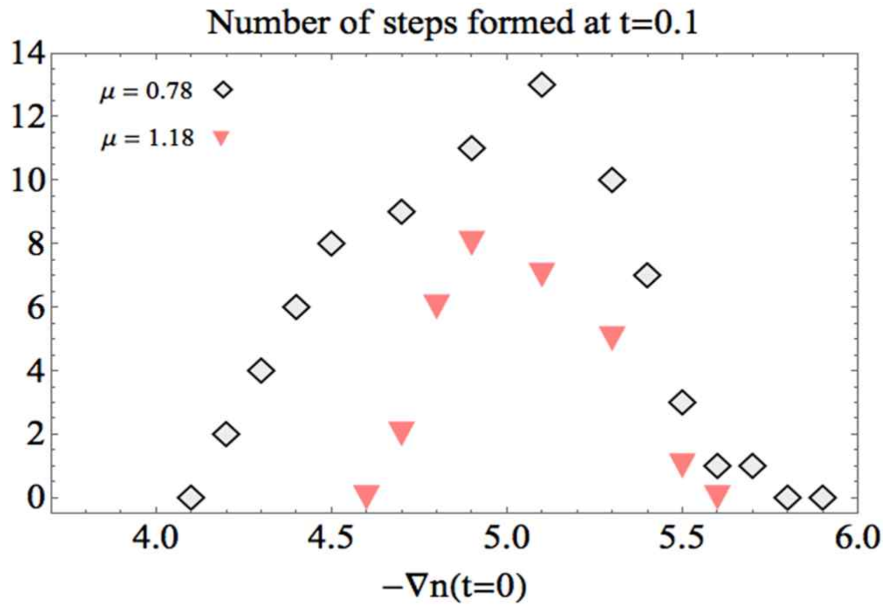
Initial condition dependence

○**Solutions are not sensitive to initial value of turbulent PE.**

○**Initial density gradient is the parameter influencing the subsequent evolution in the system.**

○**At lower viscosity more steps form.**

○**Width of density jumps grows with the initial density gradient.**



- Staircases ←→ Life

  A little t.s. smooths the roughness

  Too much t.s. makes a mess

Observations and Lessons

→ Towards a Better Model

Lessons

- A) Staircases happen
  - Staircase is 'natural upshot' of modulation in bistable/multi-stable system
  - Bistability is a consequence of mixing scale dependence on gradients, intensity ←→ define feedback process
  - Bistability effectively locks in inhomogeneous PV mixing required for zonal flow formation
  - Mergers result from accommodation between boundary condition, drive(L), initial secondary instability
  - Staircase is natural extension of quasi-linear modulational instabilty/predator-prey model → couples to transport and b.c. ←→ simple natural phenomenon

Lessons

- B) Staircases are Dynamic (GK missed, completely)
  - Mergers occur
  - Jumps/steps migrate. B.C.'s, drive all essential.
  - Condensation of mesoscale staircase jumps into macroscopic transport barriers occurs. ➔ Route to barrier transition by global profile corrugation evolution vs usual picture of local dynamics
  - Global 1^{st} order transition, with macroscopic hysteresis occurs
  - Flux drive + B.C. effectively constrain system states.

Status of Theory

- N.B.: Alternative mechanism via jam formation due flux-gradient time delay → see Kosuga, P.D., Gurcan; 2012, 2013
- a) Elegant, systematic WTT/Envelope methods miss elements of feedback, bistability

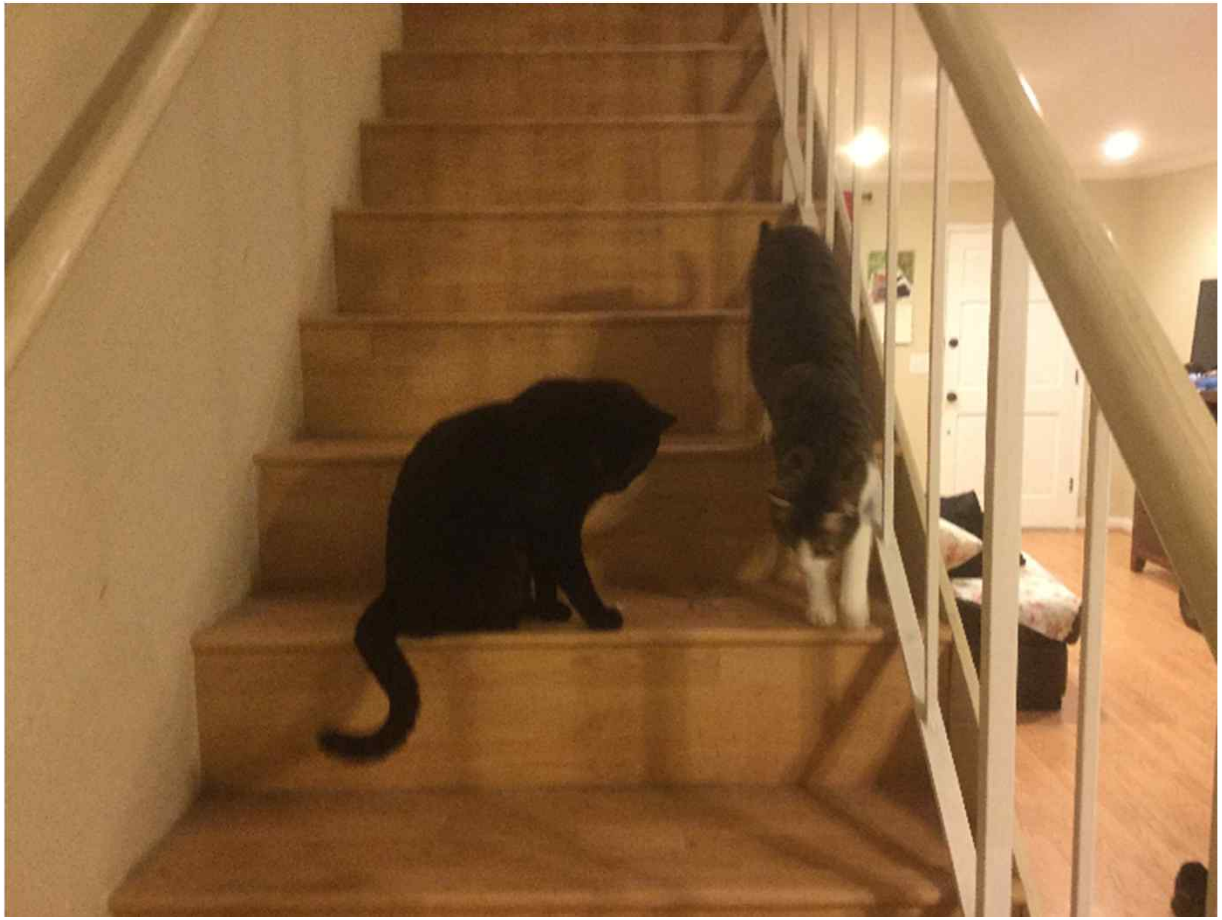
  b) $K - \epsilon$ genre models crude, though elucidate much
- Some type of synthesis needed
- Distribution of dynamic, nonlinear scales appear desirable
- Total PV conservation has demonstrated utility and leverage. Underutilized in MFE.

- Staircases appear to be:
  - Natural solution to "predator-prey" problem domains via decomposition (akin spinodal)
  - Natural reduced DOF models of profile evolution
  - Realization of 'non-local' dynamics in transport

➔ Global bifurcation via internal re-arrangement

Conclusions:

→Expect interest in staircases to increase in near future.



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